INTRODUCTION

In this chapter we shall continue with the fundamental principles of combined materials, but specifically applied to reinforced concrete, where, because of the brittle nature of concrete, certain unique considerations must be made.

The methods employed here have been traditionally referred to as working stress design methods. The current building code of the American Concrete Institute (ACI) refers to this as the alternate design method.* This method is referred to as an “alternate” because a later method, known as strength design, is stressed by the ACI Code. Although strength design (formerly called ultimate strength design) is the newest method and is winning out as a primary design procedure for reinforced concrete members, the principles employed in the working stress methods are still quite important. In this chapter we shall deal with the principles of working stress, and the methods of strength design will be presented in Chapter 15. It might be worthwhile at this point to explain, briefly, the fundamental differences in the two methods.

Working stress design treats concrete as an elastic material; members are designed much like the combined materials described in the previous chapter. Designs are based on the allowable stresses of the concrete and the reinforcing steel, and the relationship between the moduli of elasticity of the two materials. Also, the design is based on anticipated loads (live load and dead load), which are called service loads.

In strength design, the design of a structural member is based on the ultimate compressive strength of concrete and the yield strength of the rein-

*American Concrete Institute, Building Code Requirements for Reinforced Concrete, current edition.
forcing steel. The use of ultimate strength and yield strength invalidates the use of elastic theory, since these values are beyond the elastic limits of the materials. The use of these stresses implies that the member being designed will actually fail. While this is the basis for the philosophy of strength design, members, in fact, will not fail. There are a number of factors included in the design procedure that guard against this. The most noteworthy of these is called the load factor. The load factor is a multiplier by which the service loads are inflated to hypothetical design (or ultimate) loads, and it is under these inflated loads, which will not be realized in the building, that the members are designed to "fail." There are other safety factors involved in strength design; these will be discussed in studies dealing with this method.

**WORKING STRESSES AND THE "CRACKED" SECTION**

It was previously mentioned that in the working stress method, concrete is treated as an elastic material. This idea comes about because up to a certain level of stress in concrete, a plot of a stress/strain diagram, as shown in Fig. 13-1, will indicate that there is nearly a straight-line relationship between stress and strain. It is close enough, in fact, to safely say that for practical reasons, the material obeys Hooke's Law, which indicates an elastic material. Such a stress/strain relationship in concrete is determined from a compression test, since concrete exhibits no significant tensile strength. For purposes of design we say that concrete has zero tensile strength, which, of course, accounts for the development of the reinforced concrete beam where the concrete takes compression and the reinforcing steel takes the tension. It is the lack of tensile strength in concrete that forms the basis for some considerations different than those employed when dealing with sections of combined materials, where each material has compressive strength equal to the tensile strength.

Working stress methods are based on the idea that since concrete resists no tension, the section is cracked below the neutral axis, as shown in Fig. 13-2, with all of the tensile stresses concentrated in the reinforcing steel. Consequently, we consider nothing below the neutral axis, except the steel, to be part of the effective section resisting bending. A picture of the effective section for a rectangular reinforced concrete beam is shown in Fig. 13-3. The shaded portion indicates concrete in compression. This, plus the reinforcing steel, constitutes the effective section. Since the unshaded portion of the section does not carry any stress, because it is "cracked," this is not used in the computations for locating the neutral axis or in determining the transformed moment of inertia.

We'll now look at an example that may help to clarify these ideas.

**Example 13-1 (Figs. 13-4 and 13-5)**

In this example we'll determine the location of the neutral axis and the transformed moment of inertia of the effective section. To begin, we must transform the section so that we will be dealing with a homogeneous section. In reinforced concrete problems, this is done by transforming the steel to a hypothetical equivalent area of "tensile resisting concrete." The section and the necessary data are given in Fig. 13-4. It should be mentioned at this point that the modulus of elasticity of the concrete is dependent on the strength of the mix being used. Data Sheet D-23 in the Appendix gives this information, as well as the values for the modular ratio (a) based on steel having a constant modulus of elasticity of $29 \times 10^6$ k.s.i. regardless of the grade of steel being used. From this information, it can be seen that we are dealing
with concrete whose mix is such that \( f'_c = 4000 \text{ p.s.i.} \). From the Data Sheet we find that \( n = 8 \). It might be noted here that because of variations that would be found when determining the modulus of elasticity of several specimens of the same mix design, it is permissible to round off the modular ratio to the nearest whole number.

Now, with \( n = 8 \), we make the transformation by increasing the area of steel (noted as \( A_s \)) eight times, observing the rule that only the width can be increased. A point to be made here is that the thickness of the transformed area of steel is negligible, since the bars used for reinforcing are small in diameter compared to the scale of the section. We are, therefore, concerned only with the dimension to the centerline of the transformed area. It should also be noted at this time that the total depth of a concrete member is somewhat meaningless for the purposes of structural analysis. We are only concerned with the effective depth of the section, which is taken from the extreme fiber in compression to the centerline of the steel, as shown in Fig. 13–4.

Transforming the section of Fig. 13–4 by converting the steel to an equivalent area of concrete, we get the transformed section shown in Fig. 13–5, where \( A_{tr} \) is the transformed area of the reinforcing steel.

The next step is to determine the location of the neutral axis of the effective section, which is shown as the shaded portion of the section. In order to do this it must be recognized that by definition, the neutral axis (which, of course, is the centroid of the effective section) is located in a position where the \( A \) of everything on the compression side is equal to the \( A \) of everything on the tension side.

Expressed in mathematical form,

\[
A_s \bar{x}_c = A_{tr} \bar{x}_{tr}
\]

where the subscripts indicate the compression side and the tension side and \( \bar{x} \) is the distance from the centroids of the areas to the neutral axis.

To find the neutral axis of the transformed section of Fig. 13–5:

\[
12(\bar{x}) \left( \frac{x}{2} \right) = (16 \text{ in.}^2)(20 - \bar{x})
\]

\[
6x^2 = 320 - 16 \bar{x} + 2.7x - 53.3 = 0
\]

Solving the quadratic,

\[
x = \frac{-2.7 \pm \sqrt{220.5}}{2} = 6.1" 
\]

Note that there are two roots that come out of this expression. We are only concerned with the positive value.

The next step is to determine the transformed moment of inertia of the effective section with respect to the neutral axis.

\[
I_{tr} = \frac{BD^3}{12} + A_{tr}x^2
\]

where

\[
BD^3 = \frac{(12)(6.1)^3}{3} = 908 \text{ in.}^4
\]

\[
A_{tr}x^2 = (16 \text{ in.}^2)(13.9)^2 = 3091 \text{ in.}^4
\]

\[
I_{tr} = 3999 \text{ in.}^4
\]

Note that in determining the moment of inertia for the transformed steel, the transfer equation is used, which is

\[
\frac{BD^3}{12} + A_{tr}x^2
\]

However, we are considering that the transformed area of steel is very "thin"; consequently, only the \( A_{tr}x^2 \) portion of the transfer equation will contribute to the moment of inertia.

*\( f'_c \) is the notation used to indicate the ultimate compressive strength of the concrete.
Example 13–2 (Figs. 13–6 and 13–7)

In this example we’ll go a step further in our investigation of reinforced concrete members. For the given section of Fig. 13–6 and the data shown, we’ll now determine the moment-carrying capacity. First, however, it is necessary to say a few words regarding the allowable stress shown for the concrete. It was mentioned earlier that up to a certain level of stress, a concrete test specimen will exhibit a nearly straight-line relationship between stress and strain. This level of stress is somewhere close to 50% of the ultimate strength of the concrete ($f'_{c}$). The ACI Code, in fact, sets the allowable stress to be used for working stress methods at $0.45f'_{c}$ in order to keep design stresses within the range where concrete behaves nearly elastically. We will use this code requirement; therefore,

$$f_{c} = 0.45f'_{c}$$

where $f_{c}$ = allowable concrete stress

$f'_{c}$ = ultimate compressive stress

Transforming the section of Fig. 13–6, we get the transformed section shown in Fig. 13–7, with the shaded portion representing the effective section. In order to determine the moment-carrying capacity of this section based on the given allowable stresses, we must first determine the location of the neutral axis of the effective section and the moment of inertia with respect to this axis.

**Locating the neutral axis:**

$$A_{c} \bar{x}_{c} = A_{tr} \bar{x}_{tr}$$

$$\therefore \quad 10(\frac{x}{2}) = 36(18 - x)$$

This expression yields a quadratic equation; solving this, we get

$$x = 8.3''$$

**Determine the transformed moment of inertia.**

$$I_{tr} = \frac{(10)(8.3)^{3}}{3} = 1906.0 \text{ in.}^4$$

$$I_{tr} = \frac{(36)(9.7)^{3}}{3} = 3387.2 \text{ in.}^4$$

In order to determine the moment-carrying capacity of the section, we must consider the fact that we are dealing with two allowable stresses. The moment capacity of the section will be limited by the material that is at its full allowable stress. Therefore, two computations should be made.

**Concrete, $f = 1.35 \text{ k.s.i.}$**

$$M = \frac{f_{c}t}{c} = \frac{(1.35)(5293.2 \text{ in.}^4)}{(8.3')(12''')} = 71.8''$$

**Steel, $f = 20 \text{ k.s.i.}$**

$$M = \frac{f_{s}t}{c} = \frac{(20)(5293.2)}{(9.7')(9)(12''')} = 101.1''$$

Therefore, the concrete governs and the maximum moment capacity is $71.8''$, with the concrete working at its full allowable stress and the steel under its allowable.

Example 13–3 (Figs. 13–8 and 13–9)

Up to this point, the examples shown have dealt with rectangular sections. In concrete, sections can be made of virtually any shape with relative ease. The analysis of an irregular section poses no real difficulty. As an example, let’s determine the moment capacity of the T-shaped section shown in Fig. 13–8. The first step involved...
here is the location of the neutral axis. A slightly different approach is required from that used for a rectangular section. Because of the irregular shape, we must first determine if the neutral axis falls somewhere within the flange or if it falls below the flange. If the neutral axis falls within the flange, then, as far as we’re concerned for the purposes of structural analysis, we are dealing with a rectangular section with the width being that of the flange, since the concrete below the neutral axis is not part of the effective section. If, however, we determine that the neutral axis falls below the flange, then the compression zone will be irregular in shape, thus affecting the setup of the expression used to determine the neutral axis location. The method used for determining whether or not the neutral axis falls in the flange or below it is reasonably simple and makes use of the fact that the $A_x$ on both sides of the axis are equal to each other.

With this in mind, consider a reference line at the bottom of the flange and compute the $A_x$ of the tension and compression sides using the transformed section shown in Fig. 13-9.

$$A_{x_t} = (3' \times 24')(1.5') = 108 \text{ in.}^2$$

$$A_{x_c} = (32 \text{ in.}^2)(15) = 480 \text{ in.}^2$$

This shows us that with respect to the bottom of the flange, the $A_x$ of the tension side is much greater than that of the compression side, meaning that the neutral axis is below the flange. With this knowledge, we can properly set up the expression necessary to find the neutral axis. With reference to Fig. 13-9,

$$\frac{A_{x_t} - A_{x_c}}{(72 \text{ in.}^2)x + 10x} + (10x)\left(\frac{x}{2}\right) = (32 \text{ in.}^2)(15 - x)$$

$$72x + 108 + 5x^2 = 480 - 32x$$

or

$$x^2 + 20.8x - 74.4 = 0$$

Solving the quadratic, we get $x = 3.1$; the distance to the neutral axis from the top of the section is 6.1".

**Determining the transformed moment of inertia:**

$$I_x = \frac{(24)(3')^3}{12} + (72 \text{ in.}^2)(4.6)^2 = 1577.5 \text{ in.}^4$$

$$\frac{(10)(3.1)^3}{3} = 99.3 \text{ in.}^4$$

$$= \frac{(32 \text{ in.}^2)(11.9)^2}{4} = 4531.5 \text{ in.}^4$$

$$I_x = 6208.3 \text{ in.}^4$$

To determine the moment capacity of the section, we must check both materials, based on allowable stresses.

**Concrete ($f = 1.8 \text{ k.s.i.}$)**

$$M = \frac{f_{crr}}{c} = \frac{(1.8)(6208.3)}{(6.1)(12')} = 152.7^K$$

**Steel ($f = 24 \text{ k.s.i.}$)**

$$M = \frac{f_{srr}}{c} = \frac{(24)(6208.3)}{(11.9)(8)(12')} = 130.4^K$$

The steel governs, and the maximum moment-carrying capacity is 130.4^K, with the steel stressed to its full allowable and the concrete under its allowable.
THE "INTERNAL COUPLE" METHOD

To this point, the discussion and the examples have made use of the bending stress equation. There is another approach that can be taken in analyzing a reinforced concrete member, called the "internal couple" method. While in problems of analysis, the use of the bending stress equation may often be a simpler approach (especially in irregular sections), the internal couple approach is the only way to design a reinforced concrete beam. Also, an understanding of the principles involved in this approach is vital when undertaking studies in strength design methods.

First we’ll look at the ideas involved in the internal couple method, and then we’ll consider several design problems using this approach.

In order to develop the general expressions used in the internal couple method, consider the generalized section of Fig. 13-10 and the corresponding stress diagrams of Figs. 13-11 and 13-12. The principles of Chapter 7, dealing with elastic bending, are used to develop the expressions. Specifically, the forces that constitute the internal couple, shown as $R_c$ and $R$, in the figures, must be equal to each other ($\Sigma F_c = 0$); they produce the resisting moment, which must be equal and opposite to the bending moment ($\Sigma M = 0$). Expressed in equation form,

$$R_c = R_t \quad \text{and} \quad \text{B.M.} = R.M.$$

In order to expand these expressions, it should be recognized that $R_c$ is the resultant of the compressive stress, and is equal to the volume of the "wedge" of stress shown in Fig. 13-12. $R_t$ is the resultant of the tensile stress and is equal to the product of the area of steel and the stress in the steel ($f_s$), or

$$R_c = \frac{f_s B x}{2} = R_t = A_s f_s$$

and

$$\frac{f_s B x}{2} = A_s f_s \quad \text{(since} \quad R_c = R_t)$$

In order to develop an expression for the moment, take moments due to $R_c$ and $R_t$ with respect to the neutral axis.

Referring to Fig. 13-11,

$$\therefore \quad M = R(y + z)$$

which shows that the moment produced by the internal couple is the product of $R$ and the distance between the forces.

At this point it seems that a numerical example, using the internal couple method, might serve to clarify the preceding discussion.

Example 13-4 (Figs. 13-13 through 13-15)

In order to determine the moment capacity of this section of Fig. 13-13 by the internal couple method, we must first locate the neutral axis of the transformed section shown in Fig. 13-14.

$$A_c \bar{x}_c = A_t \bar{x}_t$$

(14)\((\frac{x}{2}) = 27(24 - x)\)

or

$$x^2 + 3.9x - 92.6 = 0$$
Solving the quadratic, \[ x = 7.9^\circ \]

Now, referring to the stress diagram for the transformed section, shown in Fig. 13-15, it can easily be determined which material will govern, based on given allowable stresses. This is done by using the similar triangle relationship of the stress diagram. The maximum stress allowed for the concrete is \( f_{ac} = 1.35 \text{ k.s.i.} \). The maximum stress on the steel side of the transformed section is \( f_{as} = 2.22 \text{ k.s.i.} \). Now, if the concrete is stressed to 1.35 k.s.i. at the outer fiber of the compression zone, the stress in the transformed steel is

\[
\frac{16.1}{7.9} (1.35) = 2.75 \text{ k.s.i.} > 2.22 \text{ k.s.i.}
\]

This shows that if the concrete is up to its full allowable, the steel will be stressed beyond its allowable, indicating that the steel governs. The actual stress in the concrete can be determined by the similar triangle relationship, now using the full allowable stress in the transformed steel.

\[ f_c = \frac{7.9}{16.1} (2.22) = 1.09 \text{ k.s.i.} \]

The next step is to determine the value of \( R \). Using the expression previously developed,

\[ R = A_c f_c = (3)(20 \text{ k.s.i.}) = 60 \]

The same value could have been found using the concrete side of the equation, or

\[ R = \frac{f_s B c}{2} = \frac{(1.09)(14)(7.9)}{2} = 60 \]

The moment can now be determined using the moment expression:

\[ M = R(y + z) \]

Determining the distance between the forces of the internal couple is a simple matter for this rectangular section. The force \( R_e \) will act at the center of gravity of the wedge of stress. Therefore,

\[ (y + z) = 24 - \frac{18}{3} = 24" - 2.6" = 21.4" \]

and

\[ M = \frac{60k(21.4")}{12} = k07 k \]

Note that by using the internal couple method we are able to determine the moment capacity directly from the statical relationship involved. The moment of inertia computation was not necessary; consequently, the use of this method seems simpler, but only for a rectangular section where the wedge of stress is regular and its center of gravity is easily determined. When dealing with an irregular section having an irregular wedge of stress, as shown in Fig. 13-15, the center of gravity of the wedge is not easily determined. In a case such as this, it is probably easier to compute \( f_s \) and use the bending stress equation.

**DESIGN OF REINFORCED CONCRETE BEAMS**

Thus far, we have discussed procedures necessary for the investigation of reinforced concrete beams. In an investigation problem the cross section is known, and with this information we can determine the moment capacity or load-carrying capacity and the stresses in the concrete and reinforcing steel.
Investigation problems are useful primarily for academic purposes in that such problems serve as vehicles for presenting the principles involved.

The problem usually encountered by the designer of a reinforced concrete structure is not one of investigation, but one of design. That is, the cross-sectional geometry and the area of steel must be determined in response to the known loads and maximum moments that will be produced by those loads. This means that there are many unknowns regarding the cross section, as indicated in Fig. 13-17. In fact, there are too many unknowns involved to design by mathematical processes. Fortunately, when working with reinforced concrete there are other influences that help us make some judgments regarding the physical dimensions of the cross section, leaving the determination of the area of steel to be the design problem. In any case, the section shown in Fig. 13-17 cannot be designed until the three unknowns indicated are reduced to, at most, two unknowns. The width (B) is the least critical dimension; generally, this is set based on standard form board sizes or by some predetermined architectural criterion. This leaves only two unknowns to be designed—the depth of the section and the area of steel required.

In the investigation problems previously done, it was shown that one of the materials involved governed the moment-carrying capacity of the section. If the steel or concrete was at its full allowable stress, then this was the limiting factor, and the other material was working at something less than its full allowable stress. In a design problem, however, we have the opportunity to arrange the geometry so that, for a given moment-carrying requirement, the steel and concrete will be working at their full allowable stresses. Such a section is referred to as a balanced section and, in working stress design, is considered to be ideal, since both materials are being utilized to their fullest extent. We'll now look at an example of a balanced design and discuss other considerations in the design process.

Example 13-5 (Fig. 13-18)

The section shown in Fig. 13-18 is to be designed as a balanced section to carry a moment of 70 k-ft. The width of the beam has been set at 12", as shown. In order to determine the required depth (D) and the area of steel (A_s), we must deal with the internal couple and the geometry of the stress diagram, as shown in Fig. 13-19. Since the goal is to design a balanced section, we know that the stresses in both materials are at their full allowables. Using the similar triangle relationship of the stress diagram,

\[
\frac{1.35}{x} = \frac{2.22}{D - x}
\]

and

\[
1.35D - 1.35x = 2.22x
\]

Rearranging this expression and solving for D in terms of x,

\[
1.35D - 3.57x = 0
\]

and

\[
D = 2.64x
\]  

(13-1)

We know, from the internal couple, that

\[
R_c = \frac{f'Bx}{2}
\]

and, for this problem,

\[
R_c = \frac{(1.35)(12)x}{2} = 8.1x
\]  

(13-2)
We also know, from the internal couple, that the moment is equal to the force 
$R_2$ times the distance between $R_2$ and $R_1$, or

$$M = R_2(D - x/3)$$

Substituting Eqs. (13-1) and (13-2) for $D$ and $R_2$, we get

$$70'' \times 12'' = (8.1)(2.64x - .33x) = 18.7x^2$$

and

$$x = 6.7''$$

Substituting this value into Eq. (13-1), we can find the value for $D$:

$$D = 2.64x = 2.64(6.7) = 17.7''$$

In order to find the area of steel ($A_s$), we can use Eq. (13-2) and solve for $R_c$:

$$R_c = (8.1)x = (8.1)(6.7) = 54.3''$$

We also know that

$$R_c = R_t \quad \text{and} \quad R_t = A_s f_t$$

and

$$A_s = 2.7 \text{ in.}^2$$

The design of this balanced section is now complete.

It must be emphasized, at this point, that given the geometry of the stress diagram with the values of allowable stresses for both materials, there is only one value that $D$ can be for the given moment. Any other arbitrarily chosen value for the effective depth will cause this section to be something other than balanced.

It was mentioned earlier that in a building design, often there are certain factors that will force us to set dimensions for a reinforced concrete beam. These factors can set the depth requirement. For example, architectural requirements may call for a beam from the top of an opening to the floor line, as shown in Fig. 13-20. Whatever this total depth may be, the effective depth will be about 2 in. less than this. Except by some remote possibility, this section will be something other than balanced. The design problem here, with the width and depth set, comes down to sizing the area of steel. When dealing with such a condition, where we are restrained from making the depth the unique dimension that will cause the beam to be balanced, there are two other possibilities of stress relationships that must be considered. They are referred to as an underreinforced section and an overreinforced section.

Summarizing the three possibilities and defining the expressions:

1. Balanced section. Both materials are working at their full allowable stresses. Generally not practical because of architectural requirements or standard form board dimensions which set the width and the depth of the section.

2. Underreinforced section. This is a section that is deeper than a balanced section would be for a given moment, causing the steel to be at its full allowable stress and the concrete to be below its allowable stress.

3. Overreinforced section. This is a section that is shallower than a balanced section would be for a given moment, causing the concrete to be at its full allowable stress and the steel to be below its allowable.
As mentioned previously, the balanced section is ideal, but because of practical considerations, it can rarely be achieved. Given the other two alternatives, the underreinforced section is more desirable; in fact, practically all members designed by working stress methods are designed as underreinforced members. The reason this is more desirable is because an underreinforced section provides an extra measure of safety. If for some reason a reinforced concrete beam is subjected to a serious overload, the stresses in both materials will increase accordingly. In an underreinforced section, the steel is at its full allowable stress under normal loads, and in an overload situation the stresses will approach the yield strength of the steel with corresponding large strains. Because of its ductility, the steel will not break, but the bottom side of the beam will crack seriously, thereby giving warning of failure. The beam will remain intact although seriously deflected.

In an overreinforced section, where the steel is under its allowable and concrete is at its full allowable, an overload situation may cause the concrete to reach its ultimate strength before the steel yields, thereby causing a failure in the compression zone. A compression failure in concrete is, literally, explosive in nature, and there is little advance warning. This is a dangerous sort of failure; consequently, the design of overreinforced sections is to be avoided.

Since an underreinforced section is rather common, we will now go through an example of the design of such a section.

**Example 13–6 (Figs. 13–21 and 13–22)**

For this problem we will redesign the section of Fig. 13–18, which was initially designed as a balanced section to carry a moment of 70 k. For the balanced design it was determined that a depth of 17.7" was required. For the same width and moment we will now make the section 22" deep, as shown in Fig. 13–21. This will cause the section to be underreinforced, since it is deeper than the balanced section for the given condition. Since the section is underreinforced, we know that the steel will govern and it will be at its full allowable stress, as shown in the stress diagram of Fig. 13–22. In order to design this section—which, in essence, is to size the area of steel required—we must determine the location of the neutral axis. This is the complex part of the problem. Using the geometry of the stress diagram and the statical relationships involved will provide a solution. To begin, we know that:

\[ R_c = R_s (2F_s = 0) \]

and

\[ R_c = \frac{f_c B (x)}{2} = R_s = A_s f_s \]  \hspace{1cm} (13–3)

Using the Internal Couple,

\[ M = 70 k \times 12'' = A_s f_s \left( \frac{22 - x}{3} \right) \]  \hspace{1cm} (13–4)

and, by similar triangles,

\[ \frac{f_c}{x} = \frac{2.22}{22 - x} \Rightarrow f_c = \frac{2.22x}{22 - x} \]  \hspace{1cm} (13–5)

Substituting this value into Eq. (13–3),

\[ R_c = \frac{2.22x}{22 - x} \left( \frac{1}{2} \right) (12) (x) = \frac{13.32x^2}{22 - x} \]

and substituting this value of \( R_c \) into the "concrete" side of Eq. (13–4),
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\[ 70^{*k} \times 12'' = \frac{13.32x^2}{22 - x}(22 - \frac{x}{3}) \]

and

\[ 840 = \frac{293x^2 - 4.4x^3}{22 - x} \]

which is a cubic equation whose solution is achieved most easily by a trial-and-error method. This is so because we are looking for a positive value for \( x \), and we know it must be somewhere between zero and 22", which is the depth of the section. The range of possibilities is not even that broad. From the experience of working several problems, we know that the neutral axis is several inches down and certainly nowhere close to the steel. A narrower and more likely range for \( x \) is probably somewhere between 5" and 10". After two trials the value of \( x \) for this problem was found, within acceptable limits of accuracy, to be

\[ x = 7'' \]

Now we can proceed to determine the area of steel required. Using the "steel" side of Eq. (13–4)

\[ 70^{*k} \times 12'' = A_s(20)(22 - 2.33) \]

and

\[ A_s = \frac{840}{440 - 46.6} = 2.14 \text{ in.}^2 \]

**ONE-WAY SLABS**

One-way slabs, as shown in the framing plan of Fig. 13–23, are analyzed and designed in precisely the same manner as beams. For the sake of convenience, we generally deal with a typical 1-ft-wide strip of slab, as shown in the section of Fig. 13–24. This, then, becomes a shallow beam with a width of 12". The information determined from this typical strip is then repeated for every foot of width. Since the process involved is the same as that of beam design, there is no need to present further examples.

The previous discussions and procedures made no reference to specific recommendations made by the ACI Code, which provides a great deal of input into the design process, such as minimum area of steel requirements, minimum steel coverage, and other detail items. In practice, the ACI Code should be adhered to, thereby introducing, into the design process, certain factors that have not been discussed here.
### REINFORCED CONCRETE DATA

**Values of $E$ and $n$ for Various $f'_c$**

<table>
<thead>
<tr>
<th>$f'_c$</th>
<th>$n$</th>
<th>$E_*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2500</td>
<td>10</td>
<td>2900</td>
</tr>
<tr>
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<td>9</td>
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<td>6.7</td>
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<tr>
<td>7000</td>
<td>6</td>
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*All values of $E_*$ are to be $\times 10^3$*

### REINFORCING BAR DATA

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<th>Area</th>
<th>Dia (in.)</th>
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